

Duration: 3 hours

Max. Marks: 80

N.B. (1) Question No. 1 is **COMPULSORY**.(2) Answer **ANY THREE** questions from Q.2 to Q.6.

(3) Figures to right indicate full marks.

Que. 1 a. Find Laplace Transform of $t \cos 4t \cdot \cos 7t$ $-\frac{1}{2} \frac{2s^2 - 49}{(s^2 + 16)(s^2 + 49)}$ 5b. Find Fourier series expansion of $f(x) = x$ in $(-\pi, \pi)$ $b_n = \frac{2}{\pi} \int_0^\pi x \sin nx dx = \frac{2}{\pi} \left[-\frac{x \cos nx}{n} + \frac{\sin nx}{n^2} \right]_0^\pi = \frac{2}{\pi} \left[-\frac{\pi \cos n\pi}{n} + \frac{\sin n\pi}{n^2} \right] = \frac{2}{\pi} \left[-\frac{\pi (-1)^n}{n} + 0 \right] = \frac{2(-1)^{n+1}}{n}$ 5

c. Find the orthogonal trajectory of the family of curves given by

$$2x - x^2 + 3xy^2 = a$$

$$2y + 3x^2y - y^3$$

d. If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$, Find eigen values of $A^3 - 3A^2 + 5A$ 5Que. 2 a. Obtain Fourier series expansion for $f(x) = x^2$ in $(0, 2\pi)$ $a_n = \frac{2}{\pi} \int_0^\pi x^2 \cos nx dx = \frac{2}{\pi} \left[\frac{x^2 \sin nx}{n} - \frac{2x \cos nx}{n} + \frac{2 \sin nx}{n^2} \right]_0^\pi = \frac{2}{\pi} \left[0 - \frac{2\pi \cos n\pi}{n} + \frac{2 \sin n\pi}{n^2} \right] = \frac{2}{\pi} \left[-\frac{2\pi (-1)^n}{n} + 0 \right] = \frac{4(-1)^{n+1}}{n}$ 6

By using partial fractions, find the inverse Laplace transform of

$$\frac{s^2}{(s^2+9)(s^2+16)}$$

$$A = -9/7 \quad B = 16/7$$

$$B = 16/7$$

Find the eigenvalues and the eigenvectors of the matrix

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

Que. 3 a. Find the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y + \cos 2x}$ 6b. Find the Laplace transform of $\sinh^5 t$ 6c. Using Bender Schmidt method, solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, subject to 0.2 8

$$u(0, t) = 0, \quad u(1, t) = 0, \quad u(x, 0) = \sin \pi x \quad 0 \leq x \leq 1$$

Que. 4 a. By using Laplace transform, evaluate, $\int_0^\infty \frac{\cos 3t - \cos 5t}{t} dt$ 6

Find a, b, c, d, e if

b. $f(z) = (ax^3 + bx^2y^2 + cy^3 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$ is an analytic function.Obtain the half range cosine series of $f(x) =$

$$c. \begin{cases} x & 0 < x < \pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$$

$$\frac{1}{2} \log(s^2 + 25) - \frac{1}{2} \log(s^2 + 9)$$

- Que. 5
- a. Find the analytic function $f(z) = u + iv$, in terms of z , if $u = y^3 - 3x^2y$ 6
- b. If $L\{f(t)\} = \frac{s}{s^2 + s + 4}$, find $L\{e^{-2t} f(2t)\}$ 6

- c. Determine if the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ diagonalizable, hence find its diagonal matrix D and modal matrix 6

- Que. 6
- a. Determine the Half Range Sine Series for $f(x) = \frac{x(\pi^2 - x^2)}{12}$ where $0 < x < \pi$ 6
- b. Find inverse Laplace transform of $\cot^{-1}\left(\frac{s+3}{2}\right)$ 6
- Using Crank-Nicholson simplified formula, solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ 8
- c. $u(0, t) = 0$, $u(4, t) = 0$, $u(x, 0) = \frac{x}{3}(16 - x^2)$ for one step for time.

$$h = 1$$

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6
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2
5